Geographic Knowledge: Bringing Geographers to their Sensors

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ABSTRACT

This paper proposes that knowledge about the properties and spatial relationships of real-world phenomena are dependent upon, and a consequence of, an intelligent agent's sensory experience. The *a priori* existence of objects in nature is rejected. A theory of empirical spatial and property knowledge is developed, based on sensors, and sensor responses, mapping real-world structures and their properties. This theory is applied to the geographic knowledge represented by chloropleth maps and their digital database equivalents, leading to an understanding of geographic induction and deduction.

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1. INTRODUCTION

We humans integrate sensory experience of our surroundings into a stream of consciousness. The 'reality' experienced is so 'real' that, most of the time, we remained blissfully unaware of the enormous mental task that processing sensory inputs entails. We suggest that this leads to a widespread belief that external reality already conveniently structured into objects, which we simply 'see'. For most of us, it is only when our modelling breaks down, as in so-called 'optical illusions' that we become aware of our internal processing. However, for those of us who are in the business of geographic information science and related disciplines, we need to be cognisant of how mistaken this view is, and of its potential for deflecting us from effective understandings of things spatial.

The importance of prior knowledge and learning is dramatically illustrated by the case of "Virgil", as reported by Oliver Sacks (1995). Virgil had been functionally blind since early childhood. Over the years, he had been brought up, educated, lived, and worked, blind. It was only when faced with marriage at the age of fifty, and prompted by his bride-to-be, that a review of his disability indicated that his retinas might be sufficiently intact for the removal of heavy cataracts to 'restore' his vision. It was decided that he had nothing to lose from an operation. And the operation was indeed, 'successful'. The poignant moment when the bandages were removed from Virgil's eye, is recounted by Sacks,

> The dramatic moment stayed vacant, grew longer, sagged. No cry ('I can see!') burst from Virgil's lips. He seemed to be staring blankly, bewildered, without focusing, at the surgeon, who stood before him, still holding the bandages. Only when the surgeon spoke—saying 'well?'—did the look of recognition cross Virgil's face.
>
> Virgil told me later that the this first moment he had no idea what he was seeing. There was light, there was movement, there was colour, all mixed up, all meaningless, a blur. Then out of the blur came a voice that said, 'Well?' Then, and only then, he said, did he finally realise that this chaos of light and shadow was a face—and, indeed, the face of his surgeon. (Sacks, 1995, p. 107.)

As Sacks points out, "... we have spent a lifetime *learning* to see." In the case of Virgil, he had spent a lifetime learning to *touch*. That was how he mainly sensed his surroundings. After his operation, he found he had to touch things first in order to understand what he was 'seeing'. Oliver Sacks gives a moving account of the enormous difficulties Virgil experienced in making sense of his new-found vision. Sacks also reviewed the sparse history of similar cases and found that Virgil's initial experience of vision, and his subsequent extreme difficulties in coping with sighted life, is typical.

Virgil, and others like him, had their vision restored, only to be agnosic—able to see, but not able to decipher what is seen. Agnosics have a functional low-level vision system, but lack the 'knowledge' which would allow them to make sense of their visual sensory input. The vast majority of people learn this knowledge initially in childhood, and then hone it throughout life.

It has recently been observed that modularity is widely used by living entities, and in many ways, to manage complexity. (Green et al., 2001). We suggest that human sensory experiences are 'modularised' into units, which we refer to as *concepts*, or *objects*, so saving us from being overwhelmed by the potentially infinite detail offered by reality.

This paper is aligned with those who contend that modelling is a fundamental human activity, by suggesting that senses cannot, acting alone, provide a person with a coherent perception of their surroundings; a person must construct an understanding of their surroundings (e.g. Kelly 1955, Bannister and Fransella 1986, Dennett 1991, Sayer 1992, Edelman 1993). The view adopted here is that sense organ responses are used as inputs into a person's dynamic model of "reality"; a model that corresponds with what is experienced as "consciousness." (Argued in Edelman 1989 and empirically supported in Edelman et al. 1992.) This position implies that reality is inaccessible, except through the models that intelligent entities make of it.

In reviewing issues of concern within geographic information science, Goodchild (1995) emphasises representation of "real geographic phenomena". This representation, he considers, is "of such importance that one might go so far as to argue that the greatest challenge in [geographic information science] is to find ways of building useful representations of the infinitely complex world around us in the almost absurdly limited, discrete environment of a digital computer." (Goodchild 1995: 2)

In this paper we go back to basics to examine this important issue. Consistent with the recognition of the importance of modelling reality, based upon sensory experience, as noted above, we reject the *a priori* existence real-world *objects*. Rather the pragmatic, constructed nature of our views of the real-world are acknowledged.[1] In doing this, we do not claim to develop a new theory of geographic knowledge. Rather, we investigate a number of dimensions of knowledge in an attempt to develop a sound basis for practical geographic knowledge discovery. For, if the intention is to *discover* geographic knowledge, then it is prudent to have a clear idea about what is being sought—before setting out to find it.

In this paper key concepts are developed wherever possible after an examination of first principles. While at first sight this may appear to be less than efficient, it has the great advantage of offering a firm and rigorous foundation for the development of higher-level concepts for,

> in the interests of science it is necessary over and over again to engage in the critique of … fundamental concepts, in order that we may not unconsciously be ruled by them. (Einstein 1953: xii)

This paper is structured as follows: Section Two develops a description of knowledge, in part based on Zdzislaw Pawlak's (1991) theory of knowledge. During this development the initial idea of abstract knowledge being rooted in classification is progressively extended to incorporate the dimensions of syntactic versus semantic knowledge, and extensional versus intensional knowledge. Consideration of these dimensions leads to a description of inductive learning and deductive reasoning in Section Three. Section Four relates the concepts of abstract knowledge to the properties of real-world phenomena, leading to empirical property knowledge, and establishing the dimension of abstract versus empirical knowledge. Section Five extends empirical knowledge into the spatial domain, giving property versus spatial knowledge. In Section Six, we constrain spatial and property empirical knowledge to the geographic domain, leading to geographic knowledge and therefore the possibility of geographic deduction and induction—hence providing a theoretical foundation for knowledge discovery in geographic data. Conclusions are presented in Section Seven.

2. KNOWLEDGE

In his investigation of knowledge, Zdzislaw Pawlak (1991) takes the position that, "knowledge is deep-seated in the classificatory abilities of human beings and other species. For example, knowledge about the environment is primarily manifested as an ability to classify a variety of situations from the point of view of survival in the real

[1] We certainly do not question the existence of reality, nor of its structure and regularities in space and time!

world. Complex classification patterns of sensor signals probably form fundamental mechanisms of every living being.” (Pawlak 1991: 2)

And of significance to things *spatial*, Pawlak notes that, “a robot which would be able to behave “intelligently” in an environment, exploiting sensory signals about the outer realm and its internal states must classify possible situations and act accordingly.” (Pawlak 1991: 2)

More generally, Pawlak regards a *knowledge base* as a body of knowledge as that “represents a variety of basic classification skills (e.g. according to colors, temperature, etc.) of an “intelligent” agent or group of agents (e.g. organisms or robots) which constitute the fundamental equipment of the agent needed to define its relation to the environment or itself.” (Pawlak 1991: 3)

This paper adopts the approach taken by Pawlak (1991) by defining the term ‘knowledge’ in terms of classifications of objects. The idea of abstract knowledge is now explored, commencing with a consideration of classification.

2.1 Abstract Knowledge[2]

In a fundamental sense, finding *structure*—pattern—in one’s environment consists of grouping objects in meaningful ways—yielding a sets of related objects[3]. This idea leads to a simple, but useful, definition of a *concept*. Pawlak (1991:2) defines a concept as follows:

> Suppose we are given a finite set $U \neq \varnothing$ (the universe) of objects we are interested in. Any subset $X \subseteq U$ of the universe will be called a *concept* or a *category* in *U* …. (Pawlak 1991: 2)

For example, given a set of New Zealand cities, *U* = {Napier, Whangarei, Invercargill, Auckland, Christchurch, Wellington, Nelson} then the subset {Invercargill, Whangarei} is a *concept*. So is {Christchurch, Nelson}. Note that in this context, we use the terms ‘concept’, ‘category’, and ‘class’, interchangeably.

Pawlak (1991) introduces the idea of an *abstract knowledge* being any *family of concepts* that partitions the objects of interest, *U*.[4] For instance, in the above set of cities, the partition {{Napier, Whangarei}, {Invercargill, Auckland, Christchurch, Wellington}, {Nelson}} comprises an abstract knowledge about those cities. (Note that there are 2^7 = 128 ways of partitioning seven objects, so there are potentially 128 abstract knowledges about this set of cities.)

A partitioning of objects such as this into a family of mutually exclusive sets, is equivalent to *classifying* the objects into a set of classes—i.e. concepts, as discussed above. Formally, a classification of a set of objects *U* is defined as a family of sets $C = \{X_1, X_2, \ldots, X_n\}$, such that for $i, j = 1,2,\ldots,n$, these properties apply: $X_i \subseteq U$, $X_i \neq \varnothing$, $X_i \cap X_j \neq \varnothing$, $\bigcup X_i = U$.

It is well-known that an equivalence relation *R* over a set of objects, *U*, partitions *U* into a set of equivalence classes. In an equivalence class, objects are regarded *in a particular sense and context* to be mutually indiscernible (i.e. equivalent). For instance, a car and a bus may be regarded as equivalent in the context of transport because they can both carry people from one place to another. (Clearly, *equivalent* is not the same as *identical*.)

Using these ideas, one could take a set of objects and arrange them in groups of individuals which can, in some meaningful way, be viewed as equivalent. These groups would then constitute a family of equivalence classes partitioning the object set, and thereby defining a classification. This classification constitutes an *abstract knowledge* about the object set.

In principle, it is possible to partition a set of *n* objects in 2^n ways. Each of these partitions would constitute a single *elementary* abstract knowledge. Therefore, there could be a number of elementary abstract knowledges about a given set of objects. To acknowledge this, and following Pawlak (1991), a *knowledge base* is consequently defined as the relational system $K = \langle U, \mathbf{R} \rangle$ in which $U \neq \varnothing$ is a finite set called the universe, and ***R*** is a *family* of equivalence relations (Pawlak 1991: 3). Clearly, a knowledge base is made up of a family of elementary abstract knowledges, each of which corresponds to an equivalence relation R_i in ***R***.

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[2] The following development is made somewhat less tersely and is supported by illustrative examples in Aldridge 1998.
[3] For the moment we ignore the issue as to how our objects are obtained. This is discussed later in this paper.
[4] The empty set ∅ is conveniently included as a concept.

2.2 Syntactic Knowledge

So far, this discussion of knowledge has examined the grouping of equivalent (or indiscernible) objects. According to Stevens (1951: 2), "Syntactics is the study of the relation of signs to signs." An equivalence relation *R* is a relation between signs, and these signs denote objects. Therefore *R*, and equivalently, the classification *U*/*R*, can be termed *syntactic knowledge*.

Also, since a knowledge base *K* is a relational system of equivalence relations it constitutes a *family* of syntactic knowledges about *U*.

What is more, if *R* is an equivalence relation over *U*, then the family of all equivalence classes *U*/*R* can be viewed as a classification of *U* into categories. This classification constitutes knowledge defined by *R* and is termed (following Pawlak 1991) an *R-elementary syntactic knowledge* about *U*.

2.3 Semantic Knowledge

Although they are convenient mathematically, syntactic representations are limited; they simply relate the signs denoting objects. They are semantically barren. A more meaningful—and completely equivalent—*semantic* knowledge representation is now presented. This representation has important commonalties with the tabular representation of relations in the well-known relational model (Codd 1970).

We now define an *attribute*. Given a non-empty, finite set of objects of interest *U*, a primitive attribute *a* of a knowledge representation system, is defined as the onto function $a: U \rightarrow V_a$ where V_a is a set of values of *a* (after Pawlak 1991).

Since, "Semantics is the study of the relation of signs to objects." (Stevens 1951a: 2), and the attribute function relates objects to their value "signs", the function therefore defines the semantics of those objects. The attribute function is important because it provides a means for attaching meanings to concepts.

Earlier a (syntactic) *knowledge base* was defined as relational system $K = \langle U, \mathbf{R} \rangle$ where $U \neq \varnothing$ is a finite set called the universe, and **R** is a family of equivalence relations over *U*. Using the concept of attributes, a corresponding and equivalent knowledge representation system can be defined (Pawlak 1991). Given *U*, as defined, and a non-empty finite set of attributes *A*, a *knowledge representation system* (KRS) is a pair $S = \langle U, A \rangle$.

The critical difference between a knowledge base and a knowledge representation system is that the former comprises only syntactic knowledge, while a KRS includes meaning; it is *semantic knowledge*.

Pawlak (1991) proposed that a knowledge representation system could conveniently be set out in tabular form (Table 1). In this table:

Table 1 A tabular knowledge representation system (KRS) recording four semantic knowledges about a set of objects X

Object	colour	shape	size	location
x_1	red	Circle	small	north
x_2	blue	Square	large	north
x_3	red	triangle	small	south
x_4	blue	triangle	small	south
x_5	yellow	circle	small	south
x_6	yellow	square	small	south
x_7	red	triangle	large	north
x_8	yellow	triangle	large	north

- each attribute column represents an elementary *semantic* knowledge about the universe of objects and therefore labels the *syntactic* knowledge corresponding to an equivalence relation in the knowledge base;
- the header of each column is the name of an attribute function;
- each attribute value can be viewed as a name, or description, of an elementary category; and
- each row depicts a number of characteristics for one object.

Note the striking similarity between a knowledge representation system, expressed in tabular form, and a relational database table (Codd 1970).

2.4 Extensional and Intensional Knowledge

We now examine how knowledge might be, in principle, created. This goes beyond the theory outlined in Pawlak (1991).

It is well known that there are two fundamental ways of defining sets. First, a finite set may be defined by simply listing its members, as in the concept {Auckland, Hamilton, Wellington}. This is definition by *extension*. Alternatively, any set may be defined by stating the properties of its elements, as in "Cities in the North Island of

New Zealand with a population of more than 100,000”. This is definition by *intension*. [5] We propose that, just as sets can be defined by extension, or by intension, knowledge can be defined in the same ways.

In addition, these ideas of definition by extension and intension can be combined with the concepts of syntactic knowledge and semantic knowledge introduced earlier. For an example, consider a universe *U* of New Zealand cities. Apply the following *predicates* in which $City \in U$ is a free variable :

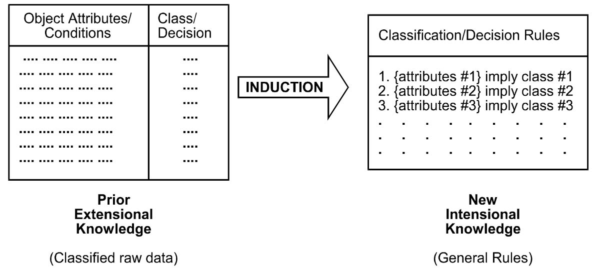

Figure 1 Supervised learning as the inductive generalisation of extensional knowledge

a) *City* is in North Island and has a population ≥ 100,000.
b) *City* is in North Island and has a population < 100,000.
c) *City* is in North Island and has a population ≥ 100,000.
d) *City* is in North Island and has a population < 100,000.

These predicates *each* define a category. *Together* they create four equivalence classes that classify (partition) *U*. The predicates therefore comprise an *intensional knowledge* about *U*. In addition the knowledge is *syntactic* because it relates signs, in this case, for city objects.

Alternatively, *semantic* intensional knowledge can be defined using production rules,

a) If *City* is in North Island and has a population ≥ 100,000 then *City* is a “big_northern_city”.
b) If *City* is in North Island and has a population < 100,000 then *City* is a “small_northern_city”.
c) If *City* is in North Island and has a population ≥ 100,000 then *City* is a “big_southern_city”.
d) If *City* is in North Island and has a population < 100,000 then *City* is a “small_southern_city”.

These rules assign values to cities and therefore *intensionally* define a “size-location” attribute function that maps *U* onto the “size-location” values (big_northern_city, etc.). These values name the four concepts in the size-location elementary knowledge. In a manner analogous to the syntactic knowledge example, *each production rule in the semantic knowledge corresponds to one concept*. And *the disjunction of the rules defines a classification of (partition on) U.*

Table 2 Knowledge-defining structures

	Syntactic	**Semantic**
Extensional	Equivalence relation/ equivalence classes	Attribute function /KRS
Intensional	Uninstantiated predicates	Production rules/ algorithms

Note that rules are therefore able to map the concepts of an existing knowledge into the concepts of a new knowledge.[6]

In summary, intensional/extensional and syntactic/semantic are independent dimensions of knowledge. Their relationships are shown in Table 2.

3. ACQUIRING KNOWLEDGE

Supervised and unsupervised learning are now introduced in terms of the knowledge theory espoused above. These concepts are particularly relevant to the later discussion of geographic knowledge and of geographic knowledge discovery in databases.

The relationship between the “facts” of extensional knowledge and the “rules” of intensional knowledge is that rules can often be viewed as *inductive generalisations* of facts. In a knowledge representation system, this generalisation can be achieved by, first, dropping the most specialised knowledge about the objects of the system, namely their identifiers. This is followed by elimination of any duplicate rows in the table.[7] This is an application of the “dropping conditions” rule of inference for syntactic generalisation in predicate calculus, as identified by Cohen and Feigenbaum (1982: 365–8). It removes the “constraint” that requires the row of a knowledge rep-

[5] I*ntension* as used in this paper should not to be confused with the similar sounding word *intention.*
[6] The rules can be viewed as *rules of inference*.
[7] If every row of the knowledge representation system is unique, there are no duplicate rows after dropping identifiers and *no inductive generalisation is achieved.* Otherwise the result is some degree of generalisation in that at least some rows refer to more than one object in *U*.

resentation system to describe a particular object. The resulting table is no longer a knowledge representation system, but can be usefully treated as an attribute decision table (c.f. Pawlak 1991: 68).

3.1 Supervised Learning

In *supervised learning*, or equivalently, *supervised classification*, the objective is to generalise from *previously classified examples*. That is, "Given a set of objects labelled by class, derive an intensional description of the classes." (op. cit.: 16). In this way, supervised learning can be viewed as *taking an extensional knowledge and transforming it into an intensional knowledge* (Figure 1). There is no new "knowledge", created. In terms of knowledge, all that is done is a transformation, from *extensional knowledge about particular instances* to *intensional knowledge about general cases*. For example, in a supervised learning session, a data table is reduced to a set of rules (c.f. Pawlak 1993) or a decision tree (c.f. Quinlan 1986).

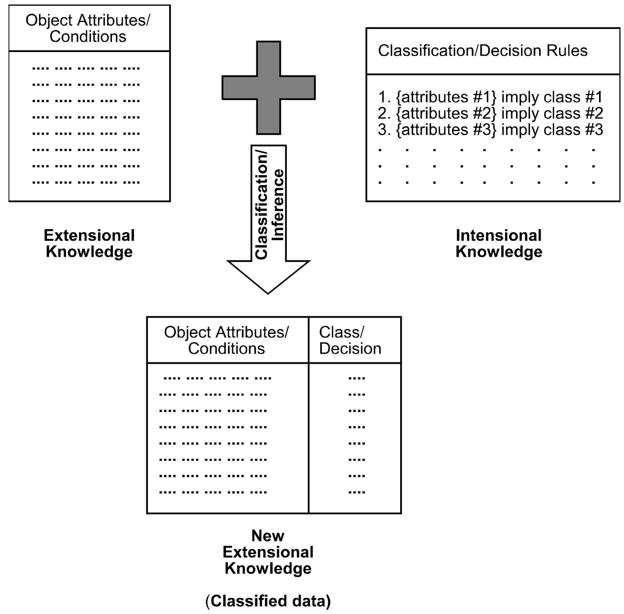

Figure 2 Using intensional knowledge to classify raw data

Once the intensional knowledge has been induced, it can be used to *deduce* new extensional knowledge from existing extensional knowledge. That is, the intensional knowledge can classify new raw data (Figure 2). This process can be described as *deductive generalisation* because the intensional knowledge is used to aggregate objects in an existing knowledge into new categories—in effect, a many-to-one functional mapping.

3.2 Unsupervised Learning

Unsupervised learning (*unsupervised classification*) does not rely on a training set; the objective is to discern knowledge—patterns—in data with minimal call on prior experience. In unsupervised learning *new intensional knowledge is created* (Figure 3). This knowledge can subsequently be used to classify additional instances having the same condition attributes.

The first stage of unsupervised learning typically draws on cluster analysis techniques for which there is a well-established literature (e.g. Everitt 1974, Richards 1993, Sonka et al. 1993). In such analysis, numeric data are evaluated in a multi-dimensional space using some sort of similarity metric. Given an appropriate metric, regularities within the data can be discerned, and the data grouped into classes. The result is a grouping of objects into more general classes or concepts. In the case of unsupervised learning, a minimum of *a priori* knowledge is used (Figure 4). This is similar to the transformation of extensional knowledge achieved during deductive inference, as in Figure 3, but without the benefit of an *a priori* intensional knowledge. Clustering therefore produces a more general *extensional* knowledge. The techniques of supervised learning can then be applied to this knowledge. This second stage results in inducing an *intensional* knowledge from the intermediate generalised extensional knowledge (c.f. Figure 1).

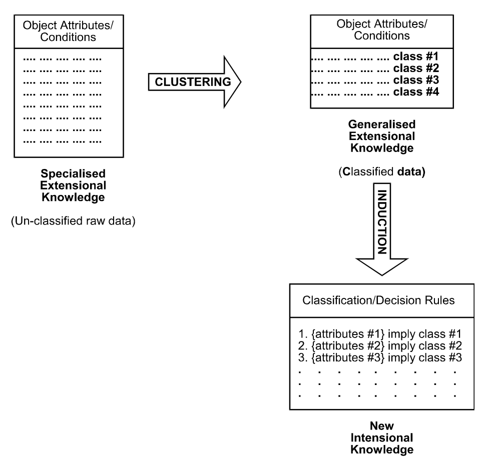

Figure 3 Unsupervised learning as induction from unclassified examples

4. EMPIRICAL PROPERTY KNOWLEDGE

We now examine the means by which an “intelligent agent” can acquire knowledge about the *properties* of its surroundings. Such knowledge is empirical in the sense that it is, “derived from or verifiable by experience, especially sense-experience.” (Brown 1993: 809). This suggests an initial definition of *empirical knowledge* as knowledge about the “external realm” of Pawlak (1991: 2), referred to earlier.

It will be shown below that the knowledge theory introduced above can be usefully applied to an understanding of: firstly, *empirical property knowledge*—knowledge about the properties of real-world phenomena; and secondly, *empirical spatial knowledge*—knowledge about the spatial relationships between such phenomena. Geographic knowledge—as represented by chloropleth maps—then becomes a particular kind of empirical knowledge.

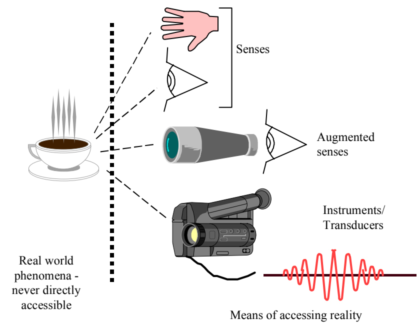

Figure 4 Sensory access to real-world phenomena

4.1 The Role of Sensors

Reality, as it is experienced by people and their machines, is in essence spatial, or more generally, spatio-temporal. It is experienced as a continuum within which individuals are able to orientate and position themselves over time. Reflecting on how reality is encountered leads to the recognition that it is never accessible directly, but only through the *interpretation* of sense or instrumental data.

To describe some real-world condition or property, senses, or sensors, must transform real-world stimuli into responses[8] that are useable by an intelligent agent (Figure 4). For instance, in humans, senses transform real-world phenomena—light, heat, pressure, etc.—into internal electro-chemical responses; ones that the neuronal structures of their brains can process.

4.2 Sensor Elements and the ‘External Realm’

Relationships between stimuli and their sensor responses are now investigated. It is contended that these relationships are the basis for acquiring knowledge about real-world phenomena. This discussion uses a charge-coupled device (CCD) array, such as is used to capture images in a digital camera, to illustrate the concepts introduced.

Sensory organs and sensors are physical entities. They have size and shape. Consequently, we now define a *sensor element* as the physical extent over which a stimulus acts when evoking a response from a sensing device.

By defining a sensor element in this way, we have an identifiable object which we can use to relate the physical world of sensors, and their responses, to the symbolic domain of object identifiers, and measures. The latter can participate in symbolic theories including, in particular, the theory of abstract knowledge introduced earlier. By using this link between the abstract and the real, the theory of abstract knowledge can be applied to empirical knowledge acquisition.

The question of relationships between sensor responses, sensor elements, and the sensor’s wider environment, is now investigated.

Firstly, the size and shape of a sensor element is a physical characteristic of the sensing system concerned. At one extreme the sensor element of a radio telescope might be a disk some thirty metres in diameter. At the other extreme, some 126 million photoreceptor cells are packed onto the retina of a human eye (Lee 1997). However, in every such case, the sensor element is the smallest spatial extent—region—to which an individual sensor response can be attributed.

Secondly, there are many possible physical relationships between sensors and the parts of reality being sensed. Thermistors, strain gauges and seismometers are directly attached to their target substrates. Many sensors are “sense-at-a-distance” devices such as microphones, electromagnetic aerials and antennas, photoelectric cells, CCD arrays, etc. These devices receive their stimulus from a distant source. The relationships between such sensors and their sensed phenomena may be described with varying degrees of sophistication, depending on the

[8] The usage of “response” in this context should not to be confused with its use in the statistical literature—as in the “response variable” of a statistical model.

extent to which influences such as reflection, refraction, attenuation, etc., need to be taken into account. Typically these relationships are described geometrically, and unique solutions are not always possible.

Figure 5 illustrates the use of a simple optical model to relate the sensors of a CCD array to its environment.

At any one time, the stimulus received by a sensor element uniquely links that element with some corresponding part of the physical world. We now define a *spatial element* ϵ_{sp} to be the part of reality which, at a particular instant, is the inferred source of the stimulus experienced by a given sensor element. The mapping between a spatial element and a sensor element is one-to-one. The spatial element is necessarily the *inferred* source of the stimulus because the sensing system needs knowledge of the relationship between the stimulus source and the sensor in order to determine adequately the relevant properties of the spatial element (see below).

Since a *sensor element* is the spatial extent over which a given sensor experiences its environment, the corresponding *spatial element* is therefore the smallest unit of 'space' that a sensor can describe. The sensor is incapable of discerning any finer structure. That is, whatever property of the spatial element is being measured, its sensor response value describes the element *as-a-whole*.

On the above basis, and disregarding for the moment the question of time-dependency, the *sensor response* corresponding to a given spatial element–sensor element coupling is concluded to be a minimal discernible experience of reality. What is more, for our "intelligent agent", each spatial element is the smallest knowable region of reality and is therefore *a fundamental spatial object*.

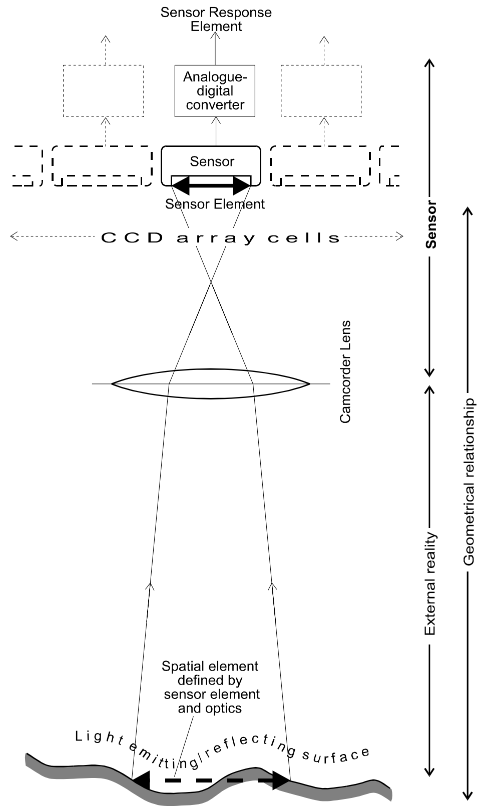

Figure 5 Camcorder CCD optics as an example of the relationship between a spatial element and a sensor response element

Clearly, the size of a spatial element depends on how a sensor element is related to the real-world phenomena being sensed. For instance, if the CCD imaging device in Figure 5 were the sensor for an astronomical telescope focused on a distant galaxy, one of the sensor elements might subtend a spatial element several thousand light years wide. On the other hand, when used with a microscope, the same imaging device might subtend a spatial element of mere microns.

4.3 Acquiring Empirical Property Knowledge from the 'External Realm'

This section investigates and formally describes a series of sensor-dependent relationships that enable the *properties* of real-world phenomena to be measured, that is, mapped into empirical property knowledge. We start with sensor responses, since these are an intelligent entity's sole source of knowledge about external reality.

4.3.1 Mapping from Physical Properties to Sensor Stimuli

Suppose the intensity of an emission originating from a spatial element is a function of the some 'interesting' physical property. This relationship is called here the *property-emission function* $f_e : P \times \mathbf{F} \to E$ where P is the range of possible property 'strengths' and **F** is a set of environmental factors such as aspect, sun elevation, ambient light intensity, etc. that influence the intensity of emission, *s*, corresponding to a given property 'strength'. For instance, in a remote sensing image, the property might be termed the element's reflectance, or its colour, or its temperature.

Consider next the point isomorphism which, by definition, exists between a spatial element and a sensor element (c.f. Figure 5). A spatial element can be viewed as the source of energy *E* experienced by its corresponding sensor as stimuli *S*. Depending on how the spatial element and its corresponding sensor element are related in space, there may or may not be a simple relationship between the energy emitted and the stimulus experienced. For instance, in the case of the example CCD cell, the intensity of light experienced by the cell's sensor element will be a function of, not only the intensity of the light emitted (or reflected) from the source spatial element, but also other factors such as absorption, atmospheric scattering and the geometrical relationship between emitter and sensor (c.f. Sabins 1987). This relationship is referred to here as the *transmission function* $f_T : E \to S$ that maps energy E emitted/transmitted/reflected from the spatial element into stimuli S received by the sensor element.

4.3.2 Mapping from Sensor Stimuli to Knowledge

To support symbolic representation and subsequent manipulation, a sensor response, which is typically in analogue form, needs to be mapped into a single numeric value—its measure. In electronic devices, this is done using an analogue to digital converter. The critical aspect of this analogue-digital conversion process is that a *range* of stimuli are mapped into a *single* numeric value. In terms of the knowledge theory presented in this paper, each sensor response value—a measurement—serves to label *an equivalence class* of stimuli. In effect, an analogue to digital converter *classifies* analogue sensor responses, thereby outputting *extensional semantic knowledge* about sensor responses, and their corresponding stimuli.

Let each equivalence class be labelled with a binary integer, v_a, possibly being the output from an analogue-digital converter . This then constitutes the sensor's output, over the time interval δt needed to make a measurement; it is a *measure* of the stimulus experienced. We identify a *sensor response function* f_S which maps stimuli received from reality by the sensor onto an ordered set of *sensor response measures*, expressed as numbers.[9] That is, $v_a = f_S(s)$.

4.3.3 Mapping Overall from Physical Properties to Knowledge

The overall mapping from spatial element properties into sensor response measures is now examined.

First, the source of the sensor stimuli E was described in terms of a property-emission function $f_e : P \times \mathbf{F} \to E$. In order to focus on the principal argument, environmental variables **F** will be assumed to be a set of parameters that are constant over the spatial extent of interest. Consequently the property-emission function is simply $f_e : P \to E$.

Second, the transmission function $f_T : E \to S$ describes a mapping between the emissions E from a spatial element and the stimuli S received by a sensor element.

Finally, the overall relationship between the stimulus acting on a sensor element and its binary output, over the time interval δt needed to make a measurement, is described in terms of a *sensor measurement function* f_M, $f_M : S \to V_a$ which maps stimuli S into sensor response measurements, V_a.

The overall mapping from a property of some real world extent (spatial element) onto its symbolic representation is therefore the composition of these three functions, that is,

$$f_E \circ f_T \circ f_M : P \to V_a$$
$$\equiv f_P : P \to V_a .$$

The function f_P is what we term the property-response function. This maps the intensity of a real-world property into a corresponding numeric sensor measurement. A set of such measurements, along with their sensor identifiers, constitutes extensional semantic knowledge about the property of interest in the "external realm"; it is therefore empirical knowledge.

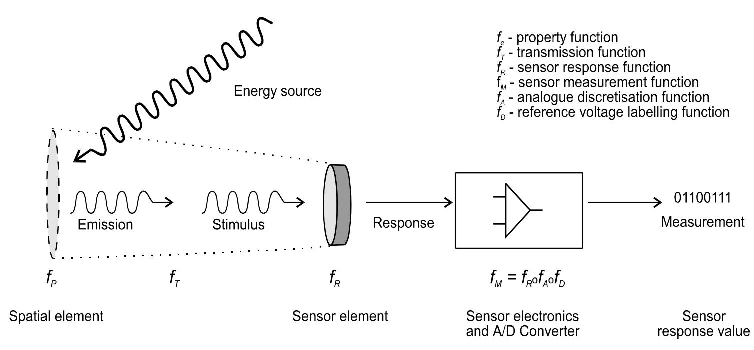

Figure 6 Conceptual relationships involved in measuring a real-world property

These mappings are illustrated in Figure 6—using a CCD cell as a example. In short, there is a functional mapping from a pre-

[9] Aldridge (1998) examines the analogue-digital functional relationships in substantially greater detail.

sumed property of reality into its symbolic representation, that is, its measurement. As a consequence, there is, at any one time, just one measurement representing that property. Together, a spatial element and its measured property constitute an intelligent entity's most fundamental 'atom' of knowledge about reality.

Notice that, so far, this discussion has not addressed possible relationships between sensor elements, and by inference, relationships between spatial elements. Empirical property knowledge is *non-spatial*.

Elementary empirical property knowledge is now defined as a knowledge representation system $S = \langle E, A \rangle$ where E is a domain of spatial elements (mapping into a set of sensor elements, R), and A is a set of attribute functions $\{a : E \to V_a\}$ having sensor response measurement domains V_a corresponding to 'strengths' of the property of interest over the spatial elements mapping into the sensor responses.[10]

Since this knowledge is expressed as a knowledge representation system (KRS), it is clearly *semantic* knowledge. It is expressed in terms of actual sensor element 'objects' and is therefore *extensional* knowledge. Finally, each sensor response measurement labels a *concept* about a real-world phenomenon.

For convenience, the term 'property knowledge', or simply 'knowledge', will be adopted when the context makes its meaning unambiguous, rather than always using the precise, but cumbersome 'semantic empirical property knowledge'. It is this knowledge that is commonly—and loosely—termed *data*.

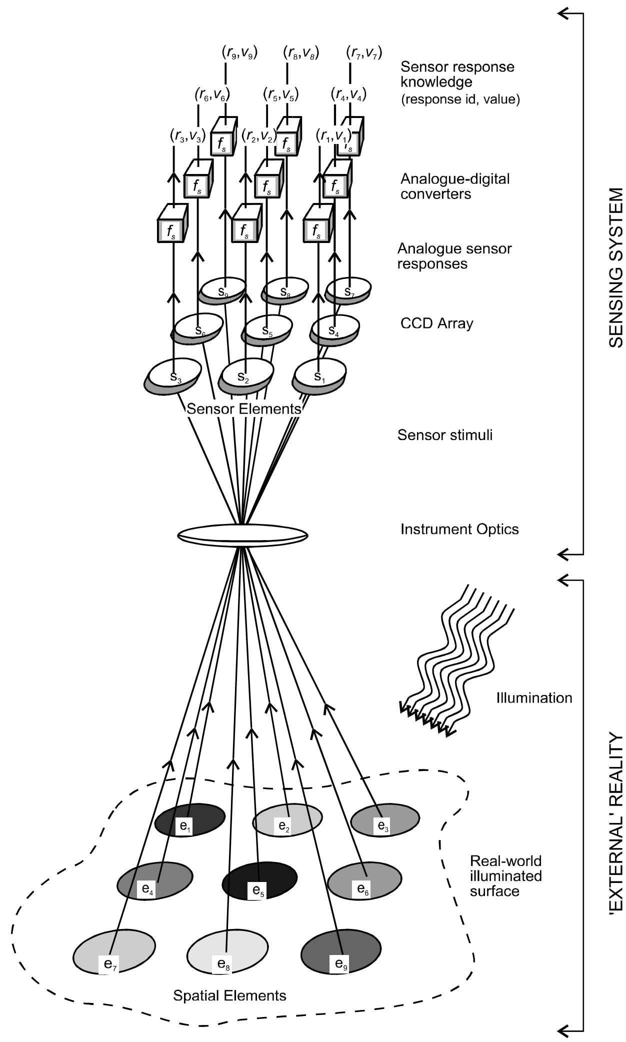

Figure 7 Structural isomorphism from spatial elements through sensor elements to sensor response elements an example sensor system

5. EMPIRICAL SPATIAL KNOWLEDGE

This section progresses from knowledge about the properties of spatial elements in isolation, to consideration of how these elements are related in space. We demonstrate that empirical spatial knowledge is derivable in this way.

This theory of spatial knowledge will later be specialised into a theory of geographic empirical knowledge, which is in turn related to the knowledge contained in chloropleth maps.

5.1 Spatial Structure

Considering now that sensors are located in physical space, there arise opportunities for an "intelligent entity" to construe additional knowledge about reality by making use of the spatial relationships between sensors. We now

[10] The significance of measures is examined in Aldridge 1998 in the context of mathematical measurement theory, in particular that of Krantz et al. (1971), whose approach relates well to the above description of sense/sensor-acquired knowledge.

investigate this using the idea of *structural isomorphism*, (a homomorphism) in which spatial relationships between element sets are preserved.

An example of the structural isomorphism from spatial elements, through sensor elements, to sensor response measurements is illustrated in Figure 8. Notice that because of the instrument optics, the grid-like arrangement (i.e. spatial structure) of the sensor elements subtends a corresponding structure (albeit laterally inverted) onto the spatial elements. Also, because of the homomorphism, connections between sensor elements and sensor response measurements, the spatial relationships between spatial elements are preserved in their property measurements.

The power of this idea of structural isomorphism is that it provides a basis for linking structures found in sensor response measurements to structures in the properties of real-world phenomena, leading to *spatial knowledge*.

5.2 Spatial Structure and Spatial Knowledge

Before discussing the issue of the symbolic representation of spatial structure, it is first noted that a symbolic system for describing structure is not always required. This is because spatial relationships between spatial elements are implicit in, and a consequence of, the configuration of the sensor elements. When higher animals process their sensor responses, these spatial relationships are preserved in the connections between the components of their massively parallel neural systems (Rumelhart and McClelland 1986). How spatial knowledge is represented and processed in the minds of humans and animals is of interest to spatial scientists but is more the research domain of cognitive psychologists.

The earlier definition of an abstract concept is now extended into the spatial domain. A *spatial concept* is a subset E_i of some universe E of spatial elements, where the spatial relationships between the elements of E_i are structurally isomorphic with a subset R_i of a universe of sensor elements R. In this way, structures in the real-world are inferred from corresponding structures in sensors.

In this theory, there are no *a priori* constraints on the number or configuration of the spatial elements that form a spatial concept. They are not required to be adjacent, contiguous, or co-planar. A spatial concept is determined by:

1. Which sensor elements are included in the concept.
2. The physical extents and configuration of these sensors.
3. The physical relationships between sensors and their isomorphic spatial elements. (In Figure 8 the instrument optics and the transmission properties of the intervening space determine this relationship.)

Following from the definition of an abstract elementary knowledge, a syntactic *elementary spatial knowledge* can now be defined as a family of spatial concepts $K = \{E_1, E_2, \ldots, E_i, \ldots, E_n\}$ that partitions the universe E of spatial elements. In the same way is for abstract knowledge, there may, of course, be many elementary spatial knowledges about any given extent E.

Our "intelligent agent" is now in a position to integrate their knowledge about the properties of, and spatial relationships between, real-world phenomena into a coherent understanding of their environment, in space and time, in terms of construed objects. How this is done is the domain of cognitive psychologists and artificial intelligence researchers, and is outside the scope of this paper.

Aldridge (1998) extends the above knowledge theory using equivalence relations between patterns of spatially related sensor elements. He shows that such an approach can provide a basis for representing spatial structure in knowledge representation systems (KRS) of the type described earlier in this paper. He also establishes that the spatial relationships between elements map into the well-known relationships between integers of order and distance (corresponding to the operators >, <, +, -, and *). A raster of spatial elements is shown to provide a basis for a 2-dimensional spatial knowledge representation system. At the same time, the development provides insight into the representation of spatial phenomena and is shown to have associations with mathematical measurement theory.

6. GEOGRAPHIC KNOWLEDGE

Concepts about abstract knowledge, empirical non-spatial knowledge and empirical spatial knowledge have been introduced. All this leads up to the focus of this paper, a theory of geographic knowledge intended to support the discovery of geographic knowledge in the digital equivalents of chloropleth maps.

In this paper, *geographic* is defined as *that which refers to locational knowledge about macroscopic phenomena at or near the surface of the earth*. This approach includes data obtained using remote sensing devices but excludes engineering, CAD, CAM data and such like. Following from this definition, it is concluded that 'geographic' is simply one concept in an application domain based categorisation of knowledges.

In geography, the traditional way of representing knowledge about the properties and spatial relationships of geographic phenomena, has been the map. Because of their pictorial nature, maps can act as surrogates for actual observation. That is, they provide alternative sources of stimuli for sensory perception. Maps attempt to preserve the spatial relations that their creators consider important. Traditionally, the relations represented have been scaled metric relations, but in recent times techniques such as multi-dimensional scaling have introduced other spatial relations as a basis for creating maps (Gatrell 1983).

Symbolic representations using Cartesian and other coordinate systems and projections are well established and are the foundation of analytical geometry. These are closely related to other geographic representations, which include rasters (Sonka et al. 1993), topological data structures (Burrough 1986), and specialised graphs such as quadtrees (Samet 1980) or adaptive recursive tessellations (Tsui and Brimicombe 1996).

The dimensions of knowledge already introduced in this paper were: abstract versus empirical; syntactic versus semantic (or attribute); extensional versus intensional; induced versus deduced; and property versus spatial. The dimension of geographic versus non-geographic has now been appended. It is now legitimate, although seriously pedantic, to refer to an induced, empirical, extensional, geographic, property knowledge. The practice is adopted herein of only using explicit terms when necessary to ensure clarity, and avoid ambiguity, and otherwise relying on context to imply appropriate knowledge dimensions.

Following from the definitions of 'geographic' and of 'spatial element', we now define a *geographic element* to be a least discernible region pertaining to macroscopic phenomenon at or near the surface of the earth. A geographic element is commonly treated as two-dimensional, but may be a 'two-and-a-half' dimensional topographic surface. The scope of this paper excludes consideration of three-dimensional or temporal phenomena. Comparison with Matheron's geostatistical "geometrical support" (Matheron 1963) is invited.

Next, we relate sensor measurements to geographic phenomena. This is done by calling on the earlier discussion of empirical spatial knowledge. Valid structural homomorphisms are therefore defined to exist *a priori* between a set of spatially related sensors and their response measurements, and a set of corresponding geographic elements and their properties.

Consider a set of sensor elements *D*, mapping into a geographic extent *E* composed of a finite set of geographic elements $E = \{e_1, e_2, \ldots, e_n\}$. Paralleling the earlier definitions of spatial concept, a *geographic concept* is defined to be a subset E_i of *E*. Observe that, as defined, a geographic concept is extensional and syntactic because it is expressed only in terms of geographic elements.

The above definition of spatial concept definition is now extended to enable the construction of complex geographic concepts using recursion. In this definition, a *geographic concept* to be either (i) a geographic element, or a (ii) a set of geographic concepts.

Following Equation 5-43 of Aldridge (1998)—and being seriously pedantic—extensional, two-dimensional elemental, geographic property knowledge is described by the knowledge representation system $G = \langle X, Y, A \rangle$. In this KRS, ordered pairs of the Cartesian product of *X* and *Y* (i.e. coordinates) identify geographic elements, which in effect are coordinated raster cells. *A* is a set of attribute functions $a: X \times Y \to V_a$ categorising data attributable to the properties geographic elements. Because the spatial elements are described by attribute values, the knowledge is semantic. The set of attribute functions might be, for instance, $A = \{land\_use, lithology, soils, slope\}$. The geographic extent covered by the knowledge is a rectangular region, the area of which is determined by the product of *cardX* and *cardY*, and by the nature of the structural isomorphism between coordinated grid elements and real-world geographic elements.

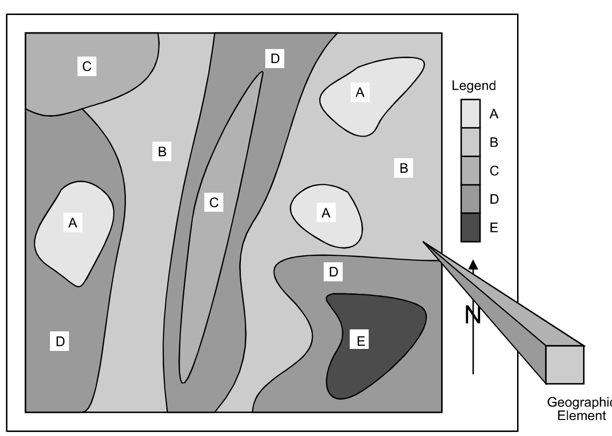

Figure 8 Chloropleth map with representative geographic spatial element

With regard to the spatial continuity of geographic concepts it is noted that, while the constituent elements of a geographic concept, are often constrained to being contiguous, such constraints are not mandatory. That is, geographic concept are not required to be continuous regions, despite this frequently being the case.

6.1 Knowledge in Thematic Maps

It is now argued that maps and their digital equivalents in geographic databases constitute extensional geographic property and spatial knowledge, and that they implicitly contain the intensional (geographic) knowledge of the map's creators: cartographers, geographers, geologists, geophysicists, etc.

Robinson et al. (1995:13) define a *thematic map* as a "... class of maps... [that] concentrates on the distribution of a single attribute or the relationship among several."[11] In such maps, the distribution of the attribute or the relationship of interest (the *theme*) may be expressed as "areas of equal value" in the form of a *chloropleth map* or as "lines connecting points of equal value" in a *isoline map* (Burrough 1986:1).

Consider a chloropleth map describing geographic extent E and partitioned into n distinct legend classes by a theme T. Then in this map:

a) Each region of equal value is a set of geographic elements E_i and is therefore a geographic concept.
b) Each legend class is a part of the geographic extent E covered by the map, i.e. $E_i \subseteq E$.
c) The areas of equal value do not overlap, i.e. $E_i \cap E_j = \varnothing$ for $i \neq j$, $i,j = 1,2,...,n$.
d) The spatial extent of the map is the sum of its areas of equal value, i.e. $E = \bigcup E_i$.
e) Every legend class has finite size, i.e. $E_i \neq \varnothing$.

Together, these subsets E_i constitute a classification of E. This classification can be expressed as a set of equivalence classes $E/T = \{E_1, E_2, \dots E_n\}$ of the equivalence relation T. Recalling the earlier definition of syntactic knowledge, the classification corresponds to the elementary syntactic knowledge D/T in which the equivalence relation T is the map's theme.

In terms of the theory presented in this paper, a chloropleth map can therefore be viewed as elementary (i.e. single-theme) knowledge about the geographic extent covered by the map. More generally, a series of chloropleth maps covering the same geographic extent E and with the family of themes $\mathbf{T} = \{T_1, T_2, \dots, T_n\}$ constitutes a basic syntactic knowledge $K = \langle E, \mathbf{T} \rangle$. This knowledge can be interpreted as corresponding to a series of physically overlaid chloropleth maps.

Consider now, the semantics of the map's legend, as applied to the map's regions. When each geographic data element in a chloropleth map is associated with a legend class value, the knowledge in the map is semantic. The chloropleth map and its legend can therefore be viewed therefore as the geographic knowledge representation system $G = \langle X, Y, \{a\} \rangle$ in which the ordered pairs of the Cartesian product of X and Y are coordinates identifying the map's geographic elements, that is, coordinate "points" in common parlance. $\{a\}$ is a single attribute $a: X \times Y \rightarrow V_a$ categorising map data elements and empirically related to the properties real-world geographic elements.

A multi-theme geographic *database* is therefore a two-dimensional, spatial knowledge representation system with multiple attribute functions $G = \langle X, Y, A \rangle$ where A is a set of map themes.

In terms of the kinds of knowledge described in this paper, a chloropleth map is made up of geographic elements and is therefore *extensional knowledge*. In creating such a map, *intensional knowledge* is used to interpolate, extrapolate, generalise and classify initially specialised empirical geographic knowledge. Stated more prosaically, when the creators of a

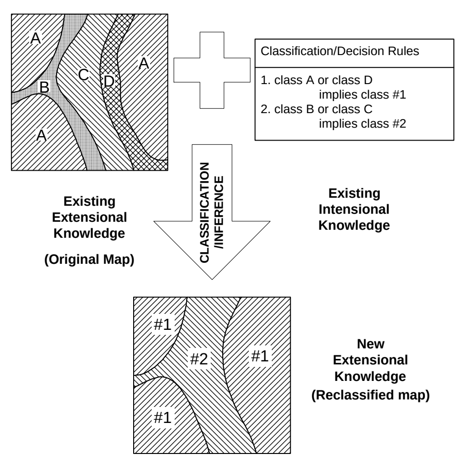

Figure 9 Reclassification as an example of geographic deduction

[11] Often one or more themes are combined with topographic and cultural overlays when a paper map is published.

chloropleth map define their map regions they are using their *a priori* intensional knowledge to classify sense or sensor data acquired from a geographic region of interest (c.f. Figure 11).

Take, for example, a topological GIS which has, say, a land-use theme (or layer, or coverage). Then each polygon in the data corresponds to only one land-use class. Similarly, a topography theme will have only one altitude class for each polygon. Thus in an *isoline map* the regions between the lines of similar value (topographic contours in this example) are classes in the theme. The above analysis of chloropleth maps therefore also applies to isoline maps, and such maps can therefore be viewed as extensional knowledge.

It has been shown that two classes of thematic maps—chloropleth maps and isoline maps—can be viewed as extensional semantic spatial knowledge constrained to the geographic domain. It therefore follows that *geographic (GIS) databases, in so far as they are digital representations of such maps, can also be regarded as extensional geographic knowledge.*

6.2 Geographic Deduction

The use of rules of inference in a deductive database to derive new knowledge from database facts has already been discussed in the context of a comparison of knowledge theory with predicate logic. It has also been argued that, in a knowledge-based view, such inference could be described as applying intentional knowledge to existing extensional knowledge so as to deduce new extensional knowledge. It has now established that thematic maps and their database equivalents represent extensional geographic knowledge. This leads to a geographic interpretation of deduction. It is proposed that *geographic deduction* occurs when intensional knowledge is applied to an existing thematic map, or its database equivalent, to create a new thematic map, as illustrated in Figure 9. In this figure, the reclassification rules take no account of spatial elements away from each target element. There is no consideration of spatially related elements in the rules shown in the figure. Such rules comprise *non-spatial* intensional geographic knowledge. By way of contrast if a rule included, say, consideration of a spatial element 200 metres west of the element under consideration, such a rule would comprise *spatial* intensional geographic knowledge. This is because the rule includes spatial relationships, namely, distance and direction. Intensional geographic knowledge can therefore be either non-spatial or spatial. This becomes an important issue when evaluating geographic knowledge discovery methods.

6.3 Geographic Induction

Geographic induction consists of acquiring intensional geographic knowledge from extensional geographic knowledge. If the resulting induced knowledge includes spatial relationships (distance, direction, contiguity, etc.) then *spatial* geographic induction has occurred. On the other hand, non-spatial geographic induction results in a *non-spatial* intensional knowledge. Significantly, *geographic induction can be regarded as the essence of geographic knowledge discovery.* What is more, geographic knowledge discovery can be either spatial or non-spatial, depending on the nature of the knowledge induced.

Non-spatial knowledge discovery requires the induction of intensional knowledge as generalisations about the properties of geographic elements. However, spatial knowledge discovery requires also the consideration of possible relationships *between* data elements. As a consequence, for a given number of elements, the search space traversed during *spatial* induction is inevitably larger than it is for *non-spatial* induction. Therefore, in principle, spatial knowledge discovery in geographic databases is a more demanding task than non-spatial knowledge dis-

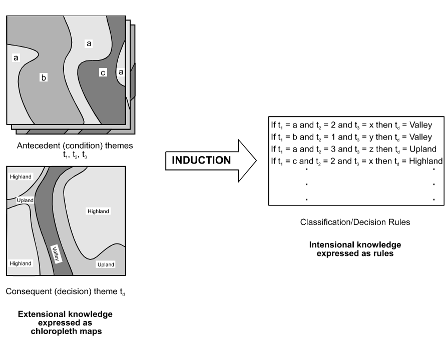

Figure 11 Supervised learning as geographic induction from extensional geographic knowledge as multiple chloropleth map themes

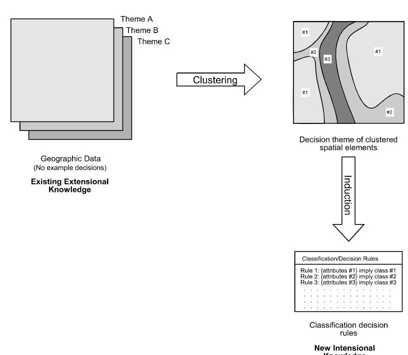

Figure 10 Unsupervised learning from extensional geographic knowledge as multiple chloropleth map themes

covery. According to Aldridge (1998) this appears to explain why a number of cases of what has been reported as spatial knowledge discovery was in fact non-spatial, being confined to the computationally less challenging element-at-a-time induction, and falling short of examining possible spatial relationships between elements.

6.3.1 Supervised Geographic Learning

In the context of geographic knowledge, supervised learning uses previously classified geographic data in the form of a thematic map (Figure 11). Supervised geographic learning as a means of discovering interesting and perhaps useful knowledge in geographic databases is further examined in Aldridge 1998.

6.3.2 Unsupervised Geographic Learning

Unsupervised geographic learning uses raw—unclassified—geographic data (Figure 10). Strictly speaking, this is geographic data in its most specialised form, where each geographic data element is a unique geographic concept, as in an unclassified remote sensing image. In a raster database this knowledge is the set of raster cells and their associated attribute values.

7. CONCLUSION

This paper extends abstract knowledge theory onto the domain empirical knowledge. Geographic knowledge is comprised of empirical spatial knowledge and empirical property knowledge, specialised to the geographic domain.

An examination of geographic knowledge leads to the conclusion that chloropleth maps, and their digital equivalents, express extensional geographic knowledge. Geographic deduction can therefore be seen as the process of using intensional geographic knowledge to transform one or more chloropleth maps into another, more generalised one. By extending ideas of inductive learning into the geographic domain, geographic induction is seen as acquiring intensional geographic knowledge from chloropleth maps, or their digital equivalents in the form of vector or raster databases, through either supervised or unsupervised learning. Geographic induction results in intensional knowledge that is either spatial or non-spatial, and as a consequence, geographic knowledge discovery may also be either spatial or non-spatial, depending on the nature of the knowledge induced. For a given data set, spatial knowledge discovery is inevitably more computationally demanding than non-spatial discovery.

We look forward to geographic information scientists gaining a clearer understanding of the nature of geographic knowledge as a result of this exploration of knowledge theory.

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