

A Wave Concept Related to the 2nd Law

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**Presented at SIRC 2004 – The 16th Annual Colloquium of the Spatial Information Research Centre
University of Otago, Dunedin, New Zealand
November 29th-30th 2004**

ABSTRACT

Existing wave concepts are based on energy conservation (1st law) and on measurable variables like wavelength, amplitude, frequency or velocity which define the energy and the behaviour of waves in space and time. A wave concept related to the 2nd law treats living systems as self-propagating wavefronts. These wavefronts are defined by the degradation of energy per unit time and by constituent sets of lineages of information which are in the most abstract case defined by positions in one spatial dimension, relatedness along the time axis, masses, energy flux shares, mass-specific metabolic rates, and adaptability; if the adaptability of the lineages is constrained, it is necessary to define the conditions in multidimensional niche space which permit lineages to propagate. The dynamics of the wavefront is defined by the likelihood and magnitude of exchanges of energy flux shares between the lineages, the macroevolutionary drift of mass and drift of mass-specific metabolic rate, the ratio of energy flux share to metabolic rate in relation to thresholds for the dissipation and amplification of lineages, and by niche drift which can be replaced by spatial drift if adaptability is unconstrained. This concept allows to create an at its roots simple numerical physics, the difference between animate and conventional inanimate physics being a higher number of relevant dimensions and the replacement of constants by measurable variables. From the physical point of view, this concept, in which the evolutionary drift is the consequence of variable natural selection in combination with constrained adaptability and genetic drift, is not restricted to any particular form of life (i.e. mechanics of information storage or processing). Cellular automata permit to apply this concept to abstract as well as realistic settings so that the causation of patterns, which are drawn from large sets of lineages and based on the variables mentioned, can be studied in one to three spatial dimensions within multidimensional niche spaces under application of arbitrarily defined rules. In application to real systems, the strategy is to determine time-independent virtual equilibria based on measurable axioms, including the assumption of energy flux conservation, and to identify neglected facts (like time dependence, demographic or microevolutionary processes/phenomena) which explain differences between the virtual, purely macroevolutionary computer worlds and reality.

Keywords and phrases: physics of life, organismic level, wave concept, energy flux conservation, cellular automata

1.0 INTRODUCTION

So far proposed wave concepts refer to the 1st law (energy is conserved). The fundamental idea behind these concepts is Huygens' principle: the geometric scheme that every point on a wavefront is the source of a new wavelet and the new wavefront is the envelope of these wavelets. It can be applied to various phenomena in the behaviour of longitudinal (amplitude) and transversal (compressional) waves, to waves in gaseous, fluid and solid media, and to electromagnetic waves. The purpose of this contribution is to point out that ideas forwarded in the 19th and 20th century allow to formulate a basic wave concept related to the 2nd law. This concept would be nothing more than an at first glance weird academic play of thought, if it gave not access to a numerical treatment of living systems at the organismic level which permits to examine the formation of macroscopic patterns based on a small but fundamental set of variables (Hashmi, 2001; 2002). Time-independent equilibria related to the assumption of energy flux conservation can be used to comprehensively examine the causation of patterns and relationships drawn from large sets of organisms. The wave concept behind this novel numerical physics is specified in more abstract terms.

2.0 THE PHYSICS OF LIVING SYSTEMS

2.1 The Physics of Life

To stay alive and keep their order, organisms have to degrade available energy, thereby steadily increase the entropy of the universe without contradicting the 2nd law (Boltzmann, 1886), which states that irreversible processes exist. In most abstract terms, life can be understood as information flux through time and matches Shannon's (1948) information concept, since consecutive generations can be seen as transmitters and receivers of germline information. According to the principle of natural selection (Darwin, 1859; Hamilton, 1964) which acts at the genetic and cultural level, organisms are heritable and mutable strategies which strive to enhance the likelihood for the transition of their own germline information through time, because information persists if it encodes an adaptive "phenome" which protects and amplifies itself.

2.2 The Maximum Power Principle and Evolutionary Constraints

Consider a system of competing, adaptive lineages in which the transition probabilities (fitness) depend on the ability to monopolize available energy. Furthermore, assume that the supply of available energy per unit time in this biosphere is finite. In a first scenario, all lineages are "Lamarckian Devils" i.e. they are unconstrained in their adaptability and know everything about the system in which they live, including the strategies of the competing lineages. Obviously, in the end, the lineages of this system degrade all available energy. Eventually, the system will be a set of persisting lineages in which extinctions are unknown, because all lineages are equally superior in their adaptability. Now consider a second scenario of so called "Darwinian Devils" which are not as diabolic as the lineages in the first scenario: although they have unconstrained adaptability, they are not aware of the decisions of the competing lineages during the next evolutionary step. In the end, also the system of Darwinian Devils degrades all available energy. Although all lineages are equally adaptive, this system knows winners and losers throughout its evolution. Once the energy supply is fully exploited, without changes of evolutionary rates, the extinction of species happens at constant rates. The decay of a set of existing species follows, like the decay of radioactive nuclei, an exponential law. In a third scenario, let there be Darwinian Devils and several forms of available energy, for example monochromatic radiation of different wavelengths. Initially, all lineages are able to degrade only one type of radiation reaching the biosphere, but with a given probability, each lineage may unintentionally attain states which allow to degrade another, so far unused, source of available energy, while natural selection favours that already acquired abilities to appropriate and use energy are maintained. If the likelihood to conquer new sources of energy is exceedingly small, the system is likely to be trapped in a state in which a single source limits the availability of energy for a long time. Also on earth evolutionary constraints prevent life from using all supplies of available energy. For example, photosynthetic organisms are unable to take advantage of the complete spectrum of degradable (Hannon and Ulanowicz, 1987) solar radiation. In conclusion, if the supply of energy to a living system is fairly constant, it is reasonable to assume that also the amount of energy degraded per unit time is constant, whether or not evolutionary constraints exist.

These ideas were qualitatively raised and sketched by Guillemint (1919) and Lotka (1922; 1925). The principle that adaptive systems maximize their energy appropriation, due to selection primarily at the level of the individual, is known as the "maximum power principle" which can be formulated as follows: if (1) the supply of available energy is constant, (2) lineages are adaptive enough, (3) the coupling of energy monopolization and fitness is strong enough and (4) the number of lineages constituting the biosphere is infinite (or large enough), then the energy appropriated and degraded by the biosphere as a whole will be constant. Otherwise it may drift.

From the empirical point of view, this principle is not without restrictions applicable to our biosphere on earth: even in pristine parts of the biosphere which have evolved without dramatic perturbations since long time, the variation of energy flux is a matter of scale, both in space and time. However, if steps in time are used which are not too small (so that, for example, subannual variation is irrelevant) and not too large (so that the evolutionary acquisition of new sources of energy is negligible) and the spatial resolution is not too fine (so that local demographic processes, like ecological succession, are unimportant), metabolic energy flux conservation is an assumption which is widely met on earth. Van Valen (1973) recognized that the decay of lineages in the paleontological record is approximately exponential. Thereby, he underlined that the Guilleminot-Lotka maximum power principle is widely applicable to macroevolutionary patterns and to some extent also to geological time scales.

3.0 A BASIC WAVE CONCEPT RELATED TO LIVING SYSTEMS

Firstly, in order to formulate a wave concept related to the 2nd law as a general physical concept, it is necessary to make sure that (1) the maximum power principle is fully applicable (energy flux conservation is the point making such a concept attractive); (2) there is no need to specify how information is stored and processed. Therefore, to enter the complexity of living systems with a basic concept, it is necessary to (1) focus on the purely macroevolutionary stage (phenomenological recognition of phenotypic changes) and to omit all demographic (birth, death, immigration and emigration) considerations as well as the microevolutionary complexity based on the attributes of genes and memes; (2) start with the scenario of “Darwinian Devils” instead of a system of widely inadaptable lineages caged in niches for which adaptation to some conditions is not possible without delay.

3.1 Essential Variables

The general physical definition of life, whether or not other forms of life beyond the one we know exist, is information flux without any specification of a genetic or cultural information carrier or the characteristics of the unfolding organism (like the number of base pairs, genes, or cells). The amount of information represented by the lineage need not be important, because small amounts of information may unfold complex developmental algorithms (which is known as the C-value paradox). However, each lineage has (1) a position in space in up to three dimensions, x , y , and z . (2) All forms of life need a physical basis to store and send the information, therefore have a metabolically active mass m which refers to the body mass of the encoded organism. (3) Lineages have to appropriate energy flux shares for maintenance. (4) The failure of losers and the replication of winners in the energetic zero-sum game consequent the growth of phylogenetic trees. The phylogenetic identity p , denotes the relatedness of a lineage at a given point in time relative to all other lineages of the same tree. It can be defined along the time axis t (SI-unit s). If exchanges and recombination of information between the lineages happen at the microevolutionary level, sexual cohesion within species may restrain the macroevolutionary variation of the parameters. Species definitions may be in accordance with relatedness denoted by p or probabilistic, if point speciations are known.

3.2 Processes as Axioms

By definition, every theory is based on axioms i.e. well documented facts which are essential for the theory and cannot be explained by the theory itself. Some of the axioms needed for the wave concept have already been specified: the fact that life is information flux and associated energy degradation, that conditions of constant energy degradation prevail, and that space and at least three additional dimensions are indispensable for the description of the state of the system of Darwinian Devils at a point in time. So far, it is not a “physical” theory, because nothing is known about the processes rendering the system dynamic. These processes have to deal with the variables already defined, refer to time, and be realistic: (1) Evolutionary rates as an exchange of energy flux shares between lineages (a “Maxwell distribution” related to the 2nd law; contrary to the related concept in classical thermodynamics formulated in 1860, the exchange rates are not predictable and have to be determined empirically). (2) An evolutionary drift of mass (which has to be calibrated empirically; for example by definition of a smallest possible mass and the likelihood to gain or loose given amounts of mass per unit time; Stanley, 1973). (3) An evolutionary drift of mass specific metabolic rate $e_m = (b m^c)/m$ (Kleiber, 1931), in which the likelihood to increase may be slightly higher than the likelihood to decrease (according to Sewertzoff 1934); b being a variable and c typically being 3/4 and falling into a range between 2/3 and 3/4 (a physical explanation for the typical value and the variation of c is given by supply-demand balance; Banavar *et al.*, 2002). (4) A random drift of Darwinian Devils in space (likelihood for a specified change in coordinates). (5) The ratio of the energy flux share e to metabolic rate e_m defines whether a lineage disappears, propagates, or splits into two identical lineages of power $e/2$. In the present concept, replication of a lineage must not be mistaken with

reproduction in the life cycle of organisms. It is a macroevolutionary process specifying spatial aspects of energy flux and permits to translate power into population sizes, densities, and the ability to distribute in space: lineages with a ratio $e/(b m^c) < 1$ dissipate; lineages may split into two lineages once they reach the threshold $e/[b(m/2)^c] = 2$ (i.e. the minimum power to split into two lineages; the threshold need not have exactly this smallest sensible value, but must not vary among the lineages of the system). The energy flux shares released by disappearing lineages are appropriated by other lineages (e.g. gained by the nearest lineage, or at equal shares assigned to lineages within a defined distance). (6) Due to sexual cohesion at the microevolutionary level, the macroevolutionary drift of variables may be uniform within sets of closely related lineages (species) defined by p . The definition of species is based on p and follows either relatedness or the likelihood of point speciation in time.

3.3 The Wave Concept

Consider a wavefront of constant power \dot{E} propagating through time in accordance with the axioms qualitatively sketched in the last paragraph. It consists of lineages of Darwinian Devils defined by space, mass, energy flux shares, and phylogenetic identity. Its dynamics is specified by the evolutionary drift of these variables plus mass-specific metabolic rates and the classification into species in accordance with phylogenetic identity. It is a true physical (mechanistic and energetic) concept based on variables and processes defined by SI-units (including species which are classified either in response to divergence along the time axis and/or point speciations happening with a given likelihood per unit time).

3.4 Extending the Concept to Systems without Constrained Adaptability

If the lineages are constrained in their ability to conquer all environmental conditions within the boundaries of the biosphere, spatial drift has to be replaced by niche drift. Niches are heritable and defined by the conditions in multidimensional niche space n which permit propagation. Spatial drift of a lineage is only allowed within its niche space. All kinds of measurable variation, including the variation created by other lineages may be relevant. In time-independent treatment, which is only applicable if the adaptability is high enough to guarantee the assumption of energy flux conservation, the local conditions always fall into the niche space of a given lineage. It is necessary to specify rules for the evolutionary drift of the niche space i.e. the increase or decrease in the number of dimensions and changes in the position and width of sections along continuous gradients or discrete classes created by these dimensions. One may consider that the niche space of a lineage may only gain or lose conditions found in the immediate vicinity of its location, or one may in addition allow a drift of the niche space into existing conditions perfectly unknown to this lineage. In the latter case preadaptation to unknown conditions is possible.

3.5 Applications

The aim of the present concept is not to predict the destiny of single lineages. It deals with macroscopic patterns i.e. variables and relationships which are drawn from large sets of lineages and may be based on all variables mentioned in section 3.1 and 3.2. Macroscopic variables are for example the number of species within a given section in space, mass distributions, range size distributions or persistence times. Relationships are associations between two such variables, for instance between body mass and evolutionary persistence time or between species richness and range size. The theory is transscalar i.e. macroscopic patterns can be calculated across all conceivable scales in the system, for example the number of species in increasing sections of space (e.g. species-area curves). The wave concept was kept as basic as possible, yet is not amenable to analytical treatment. Implementation is possible with the help of cellular automata. No distinction is made here between cellular automata and other numerical approaches. Numerical work is inherently associated with limited precision (a limited number of binary coded decimals). Cellular automata are usually defined as deterministic systems which are discrete in space and time. Basically, this definition matches numerical approaches of all kinds. Chance is non-existent in computer worlds, where pseudo-randomness is created with the help of deterministic equations. Whether the universe is seen as a system which is discrete in space and time (Wolfram 2002), or as a continuum, is not really important here. In using cellular automata to calculate macroscopic patterns, it is assumed here that it is possible to approximate a continuous reality.

How far this approach may help to understand patterns in living systems on earth has to be investigated in detail. It seems that the theory need not necessarily be tailored far to reality until it brings about macroscopic patterns which are qualitatively similar to those found in nature (Hashmi, 2002). A time-independent wave mechanics based on the present wave concept may offer the opportunity to examine pattern formation from the standpoint of a purely macroevolutionary null model. The research strategy is similar to causal inference in population genetics which aims at determining the factors that cause deviations from Hardy-Weinberg equilibria, like drift,

mutations, assortative mating, selection, immigration or emigration. The goal of the approach proposed here is to determine the factors which drive biota away from their virtual macroevolutionary equilibrium. Unfortunately, this equilibrium is not characterized by a simple equation yielding genotype frequencies. It is a set of inherently intertwined transscalar probability patterns which is calculated with the help of cellular automata and can then be compared with the corresponding patterns found in the real world. The state of the biosphere is the sum of demographic, micro- and macroevolutionary trajectories throughout its history. Factors like time-dependence, frequency dependent microevolution, multilevel selection, population cycles as implied by Lotka-Volterra dynamics in competitive systems, or drift in ecological time scales in communities without pronounced niches (Hubbell, 2001) are forces which may drive real trajectories away from their macroevolutionary, virtual equilibria. Much work remains to be done to study the robustness of the patterns in response to the variation of the parameters related to the processes specified in sections 3.2 and 3.3 and further axioms that may be added. Abstract settings are the best way to identify and catalogue effects. More realistic situations (e.g. the spatial variability of primary production on continents or along marine-terrestrial continua) allow to study why conspicuous patterns, like the latitudinal gradient of species richness, exist. Prerequisites for the examination of real systems (existing sets of organisms) are that (1) the communities and their energetics are well documented and (2) reliable phylogenetic trees for the documentation of evolutionary drift of the fundamental variables exist. In order to make this approach applicable to patterns found on earth, it is further necessary to adjust the basic concept to the prevailing conditions. Darwinian Devils and spatial evolutionary drift have to be replaced by less adaptive lineages and niche drift. Spatial scales within which the Guilleminot-Lotka maximum power principle is applicable, have to be defined empirically.

The patterns which are focal here have mainly been studied by macroecologists. The standard approach in this field of research has been to look for associations between variables with the help of bivariate or multivariate statistical methods, therefore to infer causation from existing patterns (Brown, 1995). At the given level of complexity, patterns need not inform about the underlying dynamics. Nevertheless, the macroecological approach has been important in identifying a number of conspicuous patterns and formulating qualitatively formulated hypotheses that might be responsible for the emergence of these patterns. Another field of research with a basic concept that was formulated in the 1940s has recently been explored in some detail. It recognizes that “the life cycle is the fundamental unit of description of the organism” (Caswell, 2000). This method is the exact and most appropriate way to predict the dynamics of populations in ecological time scales. However, it is not an energetic method and does not give access to a time-independent treatment. Moreover, it is too computation intensive to be applicable to communities. Therefore, also demographic theory cannot be expected to become an efficient tool to examine macroecological and -evolutionary patterns. From the microevolutionary perspective, as a third mode of inquiry, the present physics may be seen as the macroevolutionary consequence of neutral evolution at the microevolutionary level (Kimura, 1983). Neutral evolution is a mathematical concept, not a physical approach incorporating energetic reasoning. In the present concept evolution is not specified by the drift of alleles or memes. It is defined by the drift of energy flux shares which are ultimately an interplay of the genome, the phenome and the environment (Lewontin, 2000). As long as it is not possible to predict the ontogenetic and population genetic consequences of a new allele at least numerically, microevolutionary details are not of immediately relevance for the study of macroecological and -evolutionary patterns. In qualitative terms, however, microevolutionary reasoning helps to understand the macroevolutionary drift at the sub-macroevolutionary level, where the complexity of communities is too high to be treated mechanistically. Finally, it is important to underline that, as already recognized by Lotka (1921), the Guilleminot-Lotka maximum power principle is also applicable to memes at all levels of complexity brought about by cultural evolution. The concept can therefore also be applied to highly cultured living systems and economies, in which memes are crucial for the energy flux and the culturally induced cohesion at various levels of organization given by companies, ethnical groups, nations or alliances of nations competing for available energy, material resources and information. Of course, only highly complex derivatives of the basic concept formulated here, which apply more comprehensive sets of empirically defined axioms, may help to understand patterns found in cultured biospheres in detail. Nevertheless, it is possible that the number of rules required to understand political systems is not as large as one might expect, at least if the aim is to understand macroscopic patterns and underlying effects rather than historical singularities.

4.0 COMPARISON WITH CONVENTIONAL WAVE CONCEPTS

Contrary to wave concepts which have been proposed so far, the present theory refers to the 2nd law instead of the 1st law. Nevertheless, a number of similarities exist which justify to use the term “wave concept”. Its dynamics are related to a conservation law based on the Guilleminot-Lotka maximum power principle. The behaviour of wavefronts can only be understood by considering the collectivity of neighbouring self-propagating lineages. The most striking affinities exist between the present concept and atomistic wave

mechanics. The basic concept of atomistic wave mechanics is the time-independent Schrödinger-equation. It determines equilibrium states of interacting atomic nuclei and electrons under the assumption of the conservation of the sum of kinetic and potential energy (Schrödinger, 1926; Heitler, 1956). Orbitals specify exact energy levels. According to Born, they can be seen as probability patterns which denote the likelihood to find a particle within a specified integral in space. In the present concept, a species' range is a spatial representation of an exact energy flux share. In macroevolutionary terms, ranges are probability patterns which denote the likelihood to find a species within a specified integral in space. In atomistic wave mechanics, the state of the orbitals of an atom as well as the parameters defining this state (like energy, momentum, position) are inherently intertwined. The same is true for the macroscopic variables that can be determined with the help of the present wave concept. If one of those variables changes, other patterns inevitably change, too. The H-atom, as a two-body problem, is a situation which is analytically tractable by atomistic wave mechanics. Otherwise, imprecise analytical methods, like the Born-Oppenheimer approximation (which decouples the motion of the more massive nuclei from the motion of electrons) allow to treat more complex atoms and molecules. Even numerically, the time-independent Schrödinger equation cannot be solved exactly for more than 10 particles (Laughlin and Pines 2000). With respect to the present theory, it cannot be expected that analytically tractable real situations exist. However, in numerical terms, because the simulations are stochastic in nature, it is possible to simulate highly complex situations. Quantum-mechanical systems are usually seen as systems in which there is "true chance". Until this idea was forwarded early in the 20th century, chance was a mathematical concept addressing limited knowledge of details influencing macroscopic patterns (e.g. Maxwell, 1860). Alternatively, De Broglie and more comprehensively Bohm (see Bohm, 1957) thought about an alternative wave concept in which "hidden variables" are the reason for stochasticity observed in quantum mechanical systems so that apparent chance might replace the religion of true chance, which is otherwise not known in physics. I do not dare to judge about the existence of true chance. In all events, life on earth is constituted by complex quantum-systems (chemical systems). If true chance exists at the quantum level, it also persists at the higher level mechanics of interacting organisms. If it does not exist at the quantum level, the fate of individual lineages is deterministic (nevertheless not predictable in most situations). One aspect of Bohm's wave concept is that he takes into account that "hidden variables" at the sub-quantum mechanical level might influence the motion of particles, causing Brownian drift (Fig. 6 on p. 115-116 in Bohm, 1957). In the present wave concept, microevolutionary processes are the hidden variables which lead to Brownian drift of the macroevolutionary variables mentioned in sections 3.1 and 3.2.

Organisms are not the only dissipative structures in the universe that can be specified by energy degradation and the 2nd law. Tornados, hurricanes and eddies as well as stars are dissipative structures which produce entropy throughout their existence and are stable for some time. However, these structures are not self propagating in the long run for that they do not store and replicate information and are not adaptive. They channel forces by obeying immediate local constraints. In the case of stars, the local constraints are high mass and associated gravitational pressure which forces nuclear fusion and the release of nuclear energy causing the emission of widely constant amounts of radiation for some time. The concept of self-propagating waves sketched here cannot be applied to inadaptive dissipative structures. The wave concept related to the 2nd law, as outlined here, is therefore restricted to living systems.

5.0 CONCLUSIONS

A basic wave concept related to the 2nd law denotes the behaviour of dissipative structures stabilized by encoding information (i.e. living systems) which can be treated as self-propagating waves in space and time. It is the first true physical (energetic and mechanistic) concept for living systems at the organismic level which gives access to a time-independent wave mechanics that can be used as a null model to understand fundamental macroscopic patterns in the biosphere comprehensively. Abstract numerical treatments and adjustment of the basic concept to the prevailing conditions on earth may allow to further develop this numerical physics of adaptive matter which may give access to a better understanding of the biosphere.

Lotka (1922) had the vision of a canonical statistical mechanics of living systems long ago, when he recognized that the physics of life invite "... a statistical mechanics of which the units shall be, not simple material particles in ordinary reversible collision of the type familiar in the kinetic theory, collisions in which action and reaction were equal; the units in the new statistical mechanics will be energy transformers ...". More recently, two influential quantum physicists underlined that "*The central task of theoretical physics in our time is no longer to write down the ultimate equations but rather to catalogue and understand emergent behavior in its many guises, including potentially life itself. We call this physics of the next century the study of complex adaptive matter*" (Laughlin and Pines, 2000).

The concept outlined here is perhaps only one of several physical concepts which can be formulated to understand the inherent complexity of living systems at the organismic level. The attractive aspect of the wave

mechanics sketched here is that by compromising real world complexity and computational limitations it remains comprehensive.

REFERENCES

- Banavar, J.R., J. Damuth, A. Maritan and A. Rinaldo (2002) Supply-demand balance and metabolic scaling. *Proc. Natl. Acad. Sci. USA* 99, pp. 10506-10509.
- Bohm, D. (1957) Causality and chance in modern physics. Van Nostrand, Princeton.
- Boltzmann, L. (1886) Der zweite Hauptsatz der mechanischen Wärmetheorie. Gerold, Wien.
- Brown, J.H. (1995) Macroecology. University of Chicago Press, Chicago.
- Caswell, H. (2000) Matrix population models. Sinauer, Sunderland.
- Darwin, C. (1859) On the origin of species by means of natural selection or the preservation of favoured races in the struggle for life. John Murray, London.
- Guilleminot, H. (1919) La matière et la vie. Flammarion, Paris.
- Hamilton, W.D. (1964) The genetical evolution of social behaviour. I, II. *J. Theor. Biol.* 7, pp. 1-52.
- Hashmi, D. (2001) "Biodiversity Wave Mechanics": Evolution of ideal biospheres. *Abstracts of the 8th congress of the European Society for Evolutionary Biology*. University of Aarhus, 20-25 August 2001, p. 60.
- Hashmi, D. (2002) "Biodiversity wave mechanics". A physics for living systems. Cuvillier, Göttingen.
- Heitler, W. (1956) Elementary wave mechanics. Oxford University Press, Oxford.
- Hubbell, S.P. (2001) The unified neutral theory of biodiversity and biogeography. Princeton University Press, Princeton.
- Kimura, M. (1983) The neutral theory of molecular evolution. Cambridge University Press, Cambridge.
- Kleiber, M. (1931) Body size and metabolism. *Hilgardia* 6: 315-353.
- Laughlin, R.B. and D. Pines (2000) The theory of everything. *Proc. Natl. Acad. Sci. USA* 97, pp. 28-31.
- Lewontin, R. (2000) The triple helix. Harvard University Press, Cambridge.
- Lotka, A.J. (1921) Note on the economic conversion factor of energy. *Proc. Natl. Acad. Sci. USA* 7, pp. 192-197.
- Lotka, A.J. (1922) Contribution to the energetics of evolution. *Proc. Natl. Acad. Sci. USA* 8, pp. 147-151.
- Lotka, A.J. (1925) Elements of physical biology. Williams & Wilkins, Baltimore.
- Maxwell, J.C. (1860) On the motions and collisions of perfectly elastic gases. *Phil. Mag.* 4, pp. 377-409.
- Schrödinger, E. (1926) Quantisierung als Eigenwertproblem. *Annalen der Physik* 79, pp. 361-375.
- Sewertzoff, A.N. (1934) quoted in Zotin, A.I. and I. Lamprecht (1996) Aspects of bioenergetics and civilization. *J. Theor. Biol.* 180, pp. 207-214.
- Shannon, C.E. (1948) A mathematical theory of communication. *Bell System Technical Journal* 27, pp. 379-423 and 623-656.
- Stanley, M. (1973) An explanation for Cope's rule. *Evolution* 27, pp. 1-26.
- Ulanowicz, R.E. and B.M. Hannon (1987) Life and the production of entropy. *Proc. R. Soc. Lond. B* 232, pp. 181-192.

Van Valen, L. (1973) A new evolutionary law. *Evolutionary Theory* 1, pp. 1-30.

Wolfram, S. (2002) A new kind of science. Wolfram Media, Champaign.